instead of halting the progress of the student until he has mastered the fundamental concepts of the calculus, it offers him a course useful and interesting in itself where the ideas of the calculus are continually applied.

Somewhat the same purpose may be accomplished, I think, by an elementary course in differential equations. At most collegiate institutions such a course is given each year. This course may quite naturally cover a good deal of the outline proposed for the second course in calculus.

It is true in my experience with both text books and lectures that courses in differential equations divide themselves rather sharply into two classes: the first is purely formal, where the time is spent in the monotonous and, if long continued, rather deadening work of learning schemes of integration; the second consists in courses devoted to the study of existence theorems and properties of solutions. One must go to books on differential geometry or mechanics or mathematical physics to find the calculus used to state problems and the geometrical and physical notions involved further employed to aid in the solution and in the interpretation of results. It would however be necessary for the purpose at hand to include these applications of calculus under the name elementary differential equations.

The course in differential equations would include methods of integration. It should perhaps include the power series existence theorem for ordinary differential equations of the first order. There could be added many problems from geometry, mechanics and physics which are not stated as differential equations but which the student himself must formulate, solve and interpret. Such a course is a continual review of the ideas of function, continuity, derivative, integral. Families of curves, their trajectories, envelopes and singularities would be included. The plane pendulum problem gives a natural introduction to elliptic integrals and functions. The brachistochrone or some similar problem takes one at least to the Euler equation in the calculus of variations. The existence theorem mentioned or an approximation method leads to some study of Taylor's series and infinite series generally.

The student should acquire in a course of this nature considerable knowledge and technique, as well as a better understanding of the calculus. This would allow more time for the essentially slow process of the accurate statements and proofs of advanced calculus when it follows.

II. Note on the Teaching of the Principle of Mathematical Induction.

By Warren Weaver, University of Wisconsin.

The writer well remembers when he first met a proof by mathematical induction. He felt as it he had been introduced to a scientific three-shell artist. While he could find no definite loop-hole in the argument it seemed too mysterious and unreal a process to have actually effected any definite proof. The attitude of classes meeting this principle for the first time seems to justify the conclusion that this is no unusual circumstance. The fact that some ridiculously simple

illustration from one's everyday experience often wonderfully clears up hazy notions will perhaps justify the following allegory, which has been successful in bringing forth that first, wide-eyed, understanding "oh!" in several cases.

A long—indeed apparently endlessly long—line of people is seen to be standing waiting to buy tickets at a window. One wonders if some far-famed magician is to give an exhibition of extracting square roots from apparently empty silk hats. Our hero, Mr. Kueedee, being the proprietor of a rival attraction, wishes to persuade this line of people to stop waiting, and come to his show. He looks at the people, and seeing a friend, Mr. Kayplusone, he goes over to him.

"Kayplusone," he says earnestly, "I feel that you are a typical average sort of a chap. I come to you to see if I can learn the attitude which all these people take towards the proposition of coming at once to my show."

"I have indeed been thinking that matter over," replies Kayplusone, "and I will agree to come if my friend Kay ahead of me here in line will also go. And I can save you a lot of questions by telling you that mine is a typical attitude. Kay says that he, in turn, will go if the man ahead of him will."

Kueedee ponders for but a moment, and then with sudden inspiration goes to the end of the line, and is seen to be in serious conversation with the man next to the ticket window. Suddenly this man, whose name happens to be One, grabs Kueedee by the hand and calls out, "I'll do it." He speaks hurriedly to the man behind him, and starts off. This man, likewise, speaks to the man behind him, and starts off also. So that Mr. Kueedee goes, calm in the confidence that even the infinite capacity of his house is going to be taxed, and himself puts out the sign,

STANDING ROOM ONLY.

Q. E. D.

III. The Curve of Concentration for a Liquid Mixture.

By C. A. Noble, University of California.

When strong alcohol is added to weak alcohol the concentration of the mixture may be representated by the arc of a hyperbola. If, for example, to a volume v_0, of percentage-strength s_0, is added a volume v, of percentage-strength s_1, the strength of the mixture is given by

$$s = \frac{s_0 v_0 + s_1 v}{v_0 + v}.$$

Figure 1 shows the curve of concentration for the case where pure alcohol is added to a unit volume of pure water. Then $s_0 = 0$, $s_1 = 100$, $v_0 = 1$, and the equation of the curve is

$$s = \frac{100v}{1 + v}.$$